## SELF EVALUATION QUESTIONS AND ANSWERS

1 Consider the hydraulic system shown in the Figure below. The cylinder ratio is 2:1, Pressure $P_{s}=120$ bar, $A_{c}=0.002 \mathrm{~m}^{2}, A_{r}=0.001 \mathrm{~m}^{2}, F_{f}=290 \mathrm{~N}$.
i) Find the load which will cause negative pressure at the cap end of the cylinder $\&$ corresponding pressure at the rod end \& pressure drop across port $P$ to $A$ and port $B$ to $T$
ii) If the load is 5000 N , find the pressure drop across port $\mathbf{P}$ to port $\mathbf{A} \&$ corresponding pressure at the rod end. Is it possible to obtain this pressure drop valve area ratio 1:1
iv) If the valve area ratio is $2: 1$ what is the overrunning load. Comment on result


2: Consider the hydraulic circuit with resistive load shown in the Figure below. The cylinder ratio is $2: 1$, Pressure $P_{s}=120$ bar, $A_{c}=0.002 \mathrm{~m}^{2}, A_{r}=0.001 \mathrm{~m}^{2}, F_{f}=$ $500 \mathrm{~N} . \mathrm{F}_{\mathrm{L}}=5000 \mathrm{~N}$. If the valve has $\mathbf{2 : 1}$ area ratio,

Determine, $\quad P_{r} P_{\mathbf{c}}$ and total pressure drop


## Q1 Solution

Since the cylinder area is $2: 1$
$Q_{2}=\frac{Q_{1}}{2} o r\left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]=0.25$

To find the load which causes the negative pressure on cap end set $P_{c}=0$

$$
\begin{aligned}
& P_{c}=\frac{P_{s}\left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]-\left[F_{f}-F_{L}\right] / A_{r}}{\left[\frac{A_{c}}{A_{r}}\right]+\left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]} \\
& 0=P_{S}\left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]-\left[F_{f}-F_{L}\right] / A_{r} \\
& F_{L}=P_{S}\left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right] A_{r}+\left[F_{f}\right] \\
& F_{L}=120 \times 10^{5}[0.25] \times 0.001+[500]=3500 \mathrm{~N}
\end{aligned}
$$

Any load greater than 3500 N will cause a negative pressure at the cap end of the cylinder. If the overrunning load is 3500 N , the pressure at the rod end is given by

$$
\begin{aligned}
& P_{r}=\frac{P_{c} A_{c}-F_{f}+F_{L}}{A_{r}} \\
& P_{r}=\frac{0-500+5000}{0.001}=40 \mathrm{bar}
\end{aligned}
$$

The pressure drop across the port P to port A orifice is
$\Delta p_{1}=p_{s}-p_{c}=120-0=120$ bar
$\Delta p_{2}=p_{r}-p_{o}=p_{r}=40 \mathrm{bar}$
For any overrunning load greater than 3500 N , the valve will not create enough pressure drop across the port B to port A orifice to maintain control of load. Suppose the load $F_{L}=5000 \mathrm{~N}$ then we can calculate

$$
\begin{aligned}
& P_{c}=\frac{P_{S}\left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]-\frac{\left[F_{f}-F_{L}\right]}{A_{r}}}{\left[\frac{A_{c}}{A_{r}}\right]+\left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]}=\frac{120 \times 10^{5}[0.25]-\frac{[500-5000]}{0.001}}{\left[\frac{0.002}{0.001}\right]+[0.25]} \\
& =-\frac{75 \times 10^{5}}{2.25}=-33.33 \times 10^{5}=-33.33 \mathrm{bar}
\end{aligned}
$$

To create $P_{c}=-33.33 \mathrm{bar}$, the pressure drop across the port P to port A orifice must be $\Delta p_{1}=p_{s}-p_{c}=120-(-33.33)=153.33$ bar, which is not possible.

The required pressure at the rod end is

$$
\begin{aligned}
& P_{r}=\frac{P_{c} A_{c}-F_{f}+F_{L}}{A_{r}} \\
& P_{r}=\frac{-33.33 \times 10^{5} \times 0.002-500+5000}{0.001}=11.23 \mathrm{bar}
\end{aligned}
$$

Let the use the valve with the area ratio of $2: 1$
$Q_{1}=C_{d} A_{1} \sqrt{\Delta p_{1}}$
$Q_{2}=C_{d} A_{2} \sqrt{\Delta p_{2}}=C_{d} \frac{A_{1}}{2} \sqrt{\Delta p_{2}}$

Solving we get
$\frac{Q_{1}^{2}}{4 Q_{2}^{2}}=\frac{\Delta p_{1}}{\Delta p_{2}}$
$\Delta p_{1}=\frac{Q_{1}^{2}}{4 Q_{2}^{2}} \Delta p_{2}$

Using in the equation
equation $\Delta p_{1}=p_{s}-p_{c}$ becomes

$$
p_{r}=\left\{p_{s}-p_{c}\right\} \times \frac{Q_{1}^{2}}{Q_{2}^{2}}
$$

Solving for the pressure at the end of the cylinder we can get
$P_{r}=\frac{P_{c} A_{c}-F_{f}+F_{L}}{A_{r}}$
$p_{r}=\left\{p_{s}-p_{c}\right\} \times \frac{4 Q_{2}^{2}}{Q_{1}^{2}}$
$P_{c}=\frac{P_{s}\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]-\left[F_{f}-F_{L}\right] / A_{r}}{\left[\frac{A_{c}}{A_{r}}\right]+\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]}$
Setting $P_{c}=0$ we get

$$
F_{L}=P_{S} A_{r}\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]+\left[F_{f}\right]
$$

For 2:1 area ratio $Q_{2}=\frac{Q_{1}}{2}$ therefore $\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]=1$

$$
F_{L}=P_{S} A_{r}\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]+\left[F_{f}\right]=120 \times 10^{5} \times 0.001+500=12500 \mathrm{~N}
$$

Therefore $2: 1$ area ratio valve can control the overrunning load more than 2.5 times the size load controlled with a $1: 1$ area ratio valve.

Now the cap end pressure can be calculated using the equation

$$
\begin{aligned}
& P_{c}=\frac{P_{s}\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]-\left[F_{f}-F_{L}\right] / A_{r}}{\left[\frac{A_{c}}{A_{r}}\right]+\left[\frac{4 Q_{2}^{2}}{Q_{1}^{2}}\right]} \\
& P_{c}=\frac{120 \times 10^{5}[1]-\frac{[500-5000]}{0.001}}{\left[\frac{0.002}{0.001}\right]+[1]}=\frac{165 \times 10^{5}}{3}=55 \mathrm{bar}
\end{aligned}
$$

$$
\begin{aligned}
& P_{r}=\frac{P_{c} A_{c}-F_{f}+F_{L}}{A_{r}} \\
& P_{r}=\frac{55 \times 10^{5}[0.002]-500+5000}{0.001}=155 \mathrm{bar}
\end{aligned}
$$

The pressure drop across the valve is
$\Delta p_{1}=p_{s}-p_{c}=120-(55)=65$ bar,
$\Delta p_{2}=p_{r}-0=155 \mathrm{bar}$

Total pressure drop across the valve is $65+155=220$ bar

## Q2- solution

We can write the force balance on the cylinder as
$P_{c} A_{c}=F_{f}+F_{L}+P_{r} A_{r}$
Where
$F_{L}=W=$ load on the cylinder $(N)$
$F_{f}=$ frictional force ( $N$ )
solving for $P_{C}$
$P_{c}=\frac{P_{r} A_{r}+F_{f}+F_{L}}{A_{r}}$

If the area ratio is unity ( i.e. $A_{1}=A_{2}=A$ ) and the orifice equation becomes
$Q_{1}=C_{d} A \sqrt{\Delta p_{1}}$
$Q_{2}=C_{d} A \sqrt{\Delta p_{2}}$

Solving we get
$\frac{Q_{1}^{2}}{Q_{2}^{2}}=\frac{\Delta p_{1}}{\Delta p_{2}}$
$\Delta p_{1}=\frac{Q_{1}^{2}}{Q_{2}^{2}} \Delta p_{2}$
Also $\Delta p_{1}=p_{s}-p_{c}$

$$
\Delta p_{2}=p_{r}
$$

Solving we get
$p_{c}=\left\{p_{s}-p_{r}\right\} \times \frac{Q_{1}^{2}}{Q_{2}^{2}}$

$$
P_{r}=\frac{P_{s}-\left[F_{f}+F_{L}\right] / A_{c}}{\left[\frac{A_{r}}{A_{1}}\right]+\left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]}
$$

If $F_{L}=5000 \mathrm{~N}$ and other parameters as same as previous example
$P_{r}=\frac{P_{s}-\left[F_{f}+F_{L}\right] / A_{c}}{\left[\frac{A_{r}}{A_{1}}\right]+\left[\frac{\left[\hat{R}_{2}^{2}\right.}{Q_{2}^{2}}\right]}$
$P_{r}=\frac{120 \times 10^{5}-\frac{[500+5000]}{0.0020}}{\left[\frac{0.001}{0.002}\right]+[4]}=15.41 \mathrm{bar}$

Using

$$
\begin{aligned}
& P_{c}=\frac{P_{r} A_{r}+F_{f}+F_{L}}{A_{c}} \\
& P_{c}=\frac{15.41 \times 10^{5} \times 0.001+500+5000}{0.002}=38.52 \mathrm{bar}
\end{aligned}
$$

The pressure drop across the valve is

Also $\Delta p_{1}=100-38.52=61.48$ bar

$$
\Delta p_{2}=p_{r}=15.41 \mathrm{bar}
$$

$$
\Delta p_{\text {total }}=\Delta p_{1}+\Delta p_{2}=61.48+15.41=76.89 \mathrm{bar}
$$

If the valve has a $2: 1$ area ratio.

$$
\begin{aligned}
& \Delta p_{1}=\frac{Q_{1}^{2}}{Q_{2}^{2}} \Delta p_{2} \\
& P_{r}=\frac{P_{s}-\left[F_{f}+F_{L}\right] / A_{c}}{\left[\frac{A_{r}}{A_{1}}\right]+\left[\frac{Q_{1}^{2}}{4 Q_{2}^{2}}\right]} \\
& P_{r}=\frac{120 \times 10^{5}-\frac{[500+5000]}{0.002}}{\left[\frac{0.001}{0.002}\right]+[1]}=30.83 \mathrm{bar} \\
& P_{c}=\frac{P_{r} A_{r}+F_{f}+F_{L}}{A_{c}} \\
& P_{c}=\frac{30.83 \times 10^{5} \times 0.001+500+5000}{0.002}=42.92 \mathrm{bar}
\end{aligned}
$$

The pressure drop across the valve

Also $\Delta p_{1}=100-42.92=77.08$ bar

$$
\begin{aligned}
& \Delta p_{2}=p_{r}=30.83 \mathrm{bar} \\
& \Delta p_{\text {total }}=\Delta p_{1}+\Delta p_{2}=77.08+30.83=107.91 \mathrm{bar}
\end{aligned}
$$

