## **LECTURE 22 TO 23 – PROPORTIAONAL VALVES**

## SELF EVALUATION QUESTIONS AND ANSWERS

1 Consider the hydraulic system shown in the Figure below. The cylinder ratio is 2:1, Pressure  $P_s = 120$  bar,  $A_c = 0.002m^2$ ,  $A_r = 0.001m^2$ ,  $F_f = 290$  N.

i) Find the load which will cause negative pressure at the cap end of the cylinder & corresponding pressure at the rod end & pressure drop across port P to A and port B to T

ii) If the load is 5000 N, find the pressure drop across port P to port A & corresponding pressure at the rod end. Is it possible to obtain this pressure drop valve area ratio 1:1

iv) If the valve area ratio is 2:1 what is the overrunning load. Comment on result



2: Consider the hydraulic circuit with resistive load shown in the Figure below. The cylinder ratio is 2:1, Pressure  $P_s = 120$  bar,  $A_c = 0.002 \text{ m}^2$ ,  $A_r = 0.001 \text{m}^2$ ,  $F_f = 500 \text{ N}$ . If the valve has 2:1 area ratio,

Determine,  $P_r$  P<sub>c</sub> and total pressure drop



## **Q1** Solution

Since the cylinder area is 2:1

$$Q_2 = \frac{Q_1}{2} or \left[ \frac{Q_1^2}{Q_2^2} \right] = 0.25$$

To find the load which causes the negative pressure on cap end set  $P_c=0$ 

$$P_{c} = \frac{P_{s} \left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right] - \left[F_{f} - F_{L}\right]/A_{r}}{\left[\frac{A_{c}}{A_{r}}\right] + \left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]}$$

$$0 = P_{s} \left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right] - \left[F_{f} - F_{L}\right]/A_{r}$$

$$F_{L} = P_{s} \left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]A_{r} + \left[F_{f}\right]$$

$$F_{L} = 120 \times 10^{5} [0.25] \times 0.001 + [500] = 3500 N$$

Any load greater than 3500 N will cause a negative pressure at the cap end of the cylinder. If the overrunning load is 3500 N, the pressure at the rod end is given by

$$P_r = \frac{P_c A_c - F_f + F_L}{A_r}$$
$$P_r = \frac{0 - 500 + 5000}{0.001} = 40 \ bar$$

The pressure drop across the port P to port A orifice is

$$\Delta p_1 = p_s - p_c = 120 - 0 = 120 \text{ bar}$$

$$\Delta p_2 = p_r - p_o = p_r = 40 \ bar$$

For any overrunning load greater than 3500N, the valve will not create enough pressure drop across the port B to port A orifice to maintain control of load. Suppose the load  $F_L = 5000 N$  then we can calculate

$$P_{c} = \frac{P_{s} \left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right] - \frac{\left[F_{f} - F_{L}\right]}{A_{r}}}{\left[\frac{A_{c}}{A_{r}}\right] + \left[\frac{Q_{2}^{2}}{Q_{1}^{2}}\right]} = \frac{120 \times 10^{5} [0.25] - \frac{[500 - 5000]}{0.001}}{\left[\frac{0.002}{0.001}\right] + [0.25]}$$
$$= -\frac{75 \times 10^{5}}{2.25} = -33.33 \times 10^{5} = -33.33 \text{ bar}$$

To create  $P_c = -33.33$ bar, the pressure drop across the port P to port A orifice must be

 $\Delta p_1 = p_s - p_c = 120 - (-33.33) = 153.33 \text{ bar}$ , which is not possible.

The required pressure at the rod end is

$$P_r = \frac{P_c A_c - F_f + F_L}{A_r}$$
$$P_r = \frac{-33.33 \times 10^5 \times 0.002 - 500 + 5000}{0.001} = 11.23 \text{ bar}$$

Let the use the valve with the area ratio of 2:1

$$Q_1 = C_d A_1 \sqrt{\Delta p_1}$$
$$Q_2 = C_d A_2 \sqrt{\Delta p_2} = C_d \frac{A_1}{2} \sqrt{\Delta p_2}$$

Solving we get

$$\frac{Q_1^2}{4Q_2^2} = \frac{\Delta p_1}{\Delta p_2}$$

$$\Delta p_1 = \frac{Q_1^2}{4Q_2^2} \Delta p_2$$

Using in the equation

equation  $\Delta p_1 = p_s - p_c$  becomes

$$p_r = \{p_s - p_c\} \times \frac{Q_1^2}{Q_2^2}$$

Solving for the pressure at the end of the cylinder we can get

$$P_{r} = \frac{P_{c}A_{c} - F_{f} + F_{L}}{A_{r}}$$

$$p_{r} = \{p_{s} - p_{c}\} \times \frac{4Q_{2}^{2}}{Q_{1}^{2}}$$

$$P_{c} = \frac{P_{s}\left[\frac{4Q_{2}^{2}}{Q_{1}^{2}}\right] - [F_{f} - F_{L}]/A_{r}}{\left[\frac{A_{c}}{A_{r}}\right] + \left[\frac{4Q_{2}^{2}}{Q_{1}^{2}}\right]}$$
Setting  $P_{c} = 0$  we get

$$F_L = P_s A_r \left[ \frac{4Q_2^2}{Q_1^2} \right] + \left[ F_f \right]$$

For 2:1 area ratio  $Q_2 = \frac{Q_1}{2}$  therefore  $\left[\frac{4Q_2^2}{Q_1^2}\right] = 1$ 

$$F_L = P_s A_r \left[ \frac{4Q_2^2}{Q_1^2} \right] + \left[ F_f \right] = 120 \times 10^5 \times 0.001 + 500 = 12500 N$$

Therefore 2:1 area ratio valve can control the overrunning load more than 2.5 times the size load controlled with a 1:1 area ratio valve.

Now the cap end pressure can be calculated using the equation

$$P_c = \frac{P_s \left[\frac{4Q_2^2}{Q_1^2}\right] - \left[F_f - F_L\right]/A_r}{\left[\frac{A_c}{A_r}\right] + \left[\frac{4Q_2^2}{Q_1^2}\right]}$$

$$P_c = \frac{120 \times 10^5 [1] - \frac{[500 - 5000]}{0.001}}{\left[\frac{0.002}{0.001}\right] + [1]} = \frac{165 \times 10^5}{3} = 55 \text{ bar}$$

$$P_r = \frac{P_c A_c - F_f + F_L}{A_r}$$
$$P_r = \frac{55 \times 10^5 [0.002] - 500 + 5000}{0.001} = 155 \text{ bar}$$

The pressure drop across the valve is

$$\Delta p_1 = p_s - p_c = 120 - (55) = 65 \ bar,$$

$$\Delta p_2 = p_r - 0 = 155 \text{ bar}$$

Total pressure drop across the valve is 65 + 155 = 220 bar

## **Q2-** solution

We can write the force balance on the cylinder as

$$P_c A_c = F_f + F_L + P_r A_r$$

Where

$$F_L = W = load on the cylinder (N)$$
  
 $F_f = frictional force (N)$ 

solving for  $P_c$ 

$$P_c = \frac{P_r A_r + F_f + F_L}{A_r}$$

If the area ratio is unity (i.e.  $A_1 = A_2 = A$ ) and the orifice equation becomes

$$Q_1 = C_d A \sqrt{\Delta p_1}$$
$$Q_2 = C_d A \sqrt{\Delta p_2}$$

Solving we get

$$\frac{Q_1^2}{Q_2^2} = \frac{\Delta p_1}{\Delta p_2}$$
$$\Delta p_1 = \frac{Q_1^2}{Q_2^2} \Delta p_2$$

Also  $\Delta p_1 = p_s - p_c$ 

$$\Delta p_2 = p_r$$

Solving we get

$$p_{c} = \{p_{s} - p_{r}\} \times \frac{Q_{1}^{2}}{Q_{2}^{2}}$$
$$P_{r} = \frac{P_{s} - [F_{f} + F_{L}]/A_{c}}{\left[\frac{A_{r}}{A_{1}}\right] + \left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]}$$

If  $F_L = 5000 N$  and other parameters as same as previous example

$$P_{r} = \frac{P_{s} - \left[F_{f} + F_{L}\right]/A_{c}}{\left[\frac{A_{r}}{A_{1}}\right] + \left[\frac{Q_{1}^{2}}{Q_{2}^{2}}\right]}$$
$$P_{r} = \frac{120 \times 10^{5} - \frac{[500 + 5000]}{0.0020}}{\left[\frac{0.001}{0.002}\right] + [4]} = 15.41 \ bar$$

Using

$$P_c = \frac{P_r A_r + F_f + F_L}{A_c}$$

$$P_c = \frac{15.41 \times 10^5 \times 0.001 + 500 + 5000}{0.002} = 38.52 \ bar$$

The pressure drop across the valve is

Also  $\Delta p_1 = 100 - 38.52 = 61.48 \ bar$ 

 $\Delta p_2 = p_r = 15.41 \ bar$ 

 $\Delta p_{total} = \Delta p_1 + \Delta p_2 = 61.48 + 15.41 = 76.89 \ bar$ 

If the valve has a 2:1 area ratio.

$$\Delta p_{1} = \frac{Q_{1}^{2}}{Q_{2}^{2}} \Delta p_{2}$$

$$P_{r} = \frac{P_{s} - \left[F_{f} + F_{L}\right]/A_{c}}{\left[\frac{A_{r}}{A_{1}}\right] + \left[\frac{Q_{1}^{2}}{4Q_{2}^{2}}\right]}$$

$$P_{r} = \frac{120 \times 10^{5} - \frac{[500 + 5000]}{0.002}}{\left[\frac{0.001}{0.002}\right] + [1]} = 30.83 \ bar$$

$$P_{c} = \frac{P_{r}A_{r} + F_{f} + F_{L}}{A_{c}}$$

$$P_c = \frac{30.83 \times 10^5 \times 0.001 + 500 + 5000}{0.002} = 42.92 \text{ bar}$$

The pressure drop across the valve

Also  $\Delta p_1 = 100 - 42.92 = 77.08 \ bar$ 

 $\Delta p_2 = p_r = 30.83 \ bar$ 

 $\Delta p_{total} = \Delta p_1 + \Delta p_2 = 77.08 + 30.83 = 107.91 \ bar$